

Zero dimensional finite-time thermodynamic, three zones numerical model of a generic Stirling and its experimental validation

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ABSTRACT

A novel zero dimensional finite-time thermodynamic, three zones (compression, expansion and regenerator volumes) model of a generic Stirling engine has been developed. Time-dependant heat transfers and losses are considered in the three zones and with the surrounding. The model calculates the evolution of gas temperature, mass and pressure in each zone. Parametric studies and optimization of the engine are facilitated due to easy change of multiple engine parameters. Experimental validation of the model and calibration of transfer coefficients are carried out over a large range of temperature and rotation speed through comparison with a scaled instrumented engine for different geometrical configurations.

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1. Introduction

Ways for sustainable mobility and rational energy use are at the very core of research efforts towards lower green house effect gases and use of alternative energy resources. A first topic on the development of such solutions concern more efficient energy converters. A second topic often less attended to concerns waste heat recovery [1] from energy converters and use of low-quality energy renewable resources. An interesting energy converter for both objectives is the Stirling engine [2,3] as it has the potential to provide high levels of efficiency and shows the potential to use energy sources at low temperature or with low-heating values. Its development has suffered for years from the extensive use of oil in internal combustion engines as low-cost available energy.

In mobility solutions the path toward hybrid and electric vehicles opens possibilities for constant speed energy converters as main energy sources of power train [4] (after mechanical/electrical conversion), or as range extenders or emergency groups. In this perspective the efficiency/weight balance of a thermal engine is the main challenge facing future developments [2]. For stationary solutions the possibility of the use of exhaust heat of existent

devices, the use of locally available non-refined fuels [5], and the use of low temperature sources are of main interest. Stirling engines must be studied in the perspective of such challenges, incorporating innovative technologies on materials and design.

The aim of this work is to develop a model complete enough to simulate the effects of the change of a given set of parameters on efficiency and weight, but simple enough to allow exhaustive optimization of a set of parameters. The main objective of the model in the frame of this study is to provide a tool able to carry predictive trends of engine performance. As such, it will be used to optimize the characteristics of an engine corresponding to these constraints by implementing optimization algorithms calling the model to deliver the set of parameters (surfaces, size, stroke, kinematics, etc.) that corresponds the best to demand. With such a tool one can evaluate the expected gain of a quantified change on a given parameter, whether physical or design-related, and give a direction for technological improvement efforts.

After a short review of existing approaches, the model and its innovative aspects are introduced in the first section. Then, an experimental device giving the possibility of model validation over a large range of parameters is shown. Results, presented in terms of pressure and power, show good model accuracy.

2. Model

Modelling efforts on Stirling engines have been carried by a considerable number of researchers in the past and as a device its

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Nomenclature			
A	wall area in contact with the gas (m ²)	T	temperature (K)
cv	specific heat at constant volume (J/kg/K)	t	time (s)
D	diameter of a volume (m)	u	internal energy (J/kg)
h	enthalpy (J/kg)	V	volume (m ³)
h_{conv}	convective exchange coefficient (W/m ²)	v	average gas speed (m/s)
L	length of a volume (m)	W	work (J)
m	mass (kg)	γ	polytropic coefficient
N	engine speed (rpm)	λ	heat conductivity (W/m)
Nu	Nusselt number	μ	efficiency
P	pressure (Pa)	ν	kinematic viscosity (m ² /s)
P_w	power (W)	Subscript	
Pr	Prandtl number	c	cold side
Q	heat transferred energy (J)	cor	corrected
r	specific gas constant (J/kg/K)	h	hot side
Re	Reynolds number	i, j	zone
S	area (m ²)	in	bring to engine
		w	wall

functioning seems to contain no unknowns. The first who has done a numerical zero dimensional model for Stirling engine was G. Schmidt [6] which takes into account dead volume in the engine but not the imperfect regenerator. Since Schmidt, a lot of models have been developed [7–13] with different axes of work. Some take into account dead volumes [10], or imperfect regenerator [9], others consider that each volume is isothermal [11,12], or exchangers are isothermal and compression and expansion rooms are adiabatic [13].

Differences between this model and others concern in first exchangers which are not isothermal, unlike the ones of Abbas in Ref. [14] and Kongtragool in Ref. [15], but are time-dependant on three different zones with a resolution in finite time [16]. Secondly we choose to take only three zones, unlike Ref. [17], but we cut up the wall of each zone in function of the geometry (cylinder, exchanger...). Thirdly we considered the regenerator as a heat exchanger between its internal wall and the gas so the model takes into account its ability to store and release a part of the heat intrinsically, without the need to use an arbitrary efficiency coefficient like in Ref. [15]. The three zones correspond to volumes exchanging heat with a hot source (zone 1), a cold source (zone 3), and a regenerator (zone 2). Heat exchanges, heat losses as well as leaks can be affected to all three zones. A working gas describes a cycle determined by a given (but not fixed) kinematic of pistons or moving surfaces transferring gases from one zone to another. Gases are considered as ideal [18] and thermodynamic variables are considered homogenous in each volume, Fig. 1

The equation of state for ideal gases is derived for each zone equation (1).

$$P_i \cdot dV_i + V_i \cdot dP_i = m_i \cdot r \cdot dT_i + r \cdot T_i \cdot dm_i \quad (1)$$

Where P_i is the pressure in the zone “i”, V_i its volume, m_i its mass, T_i its temperature and r is the specific gas constant.

Energy conservation applies to each zone equation (2).

$$u_i \cdot dm_i + cv_i \cdot dT_i \cdot m_i = -P_i \cdot dV_i + \sum h_j \cdot dm_j + \delta Q_i \quad (2)$$

Where u_i is the internal energy, cv_i is the specific heat at constant volume, h_j the enthalpy of the gas through the interface “j”, δQ_i is the heat transferred to the gas.

Mass conservation is considered from one zone to another through a section of given diameters depending on the engine geometry. The section between zone 1 and zone 2, and between zone 2 and zone 3 are considered like diaphragm and, in function of

the pressure in each side, the mass flow is calculated from equation (3), and leaks from equation (4).

$$dm_i = S_i \cdot \frac{P_j}{\sqrt{r \times T_j}} \left(\frac{P_i}{P_j} \right)^{\frac{1}{\gamma}} \sqrt{\frac{2 \cdot \gamma}{\gamma - 1} \left(1 - \left(\frac{P_i}{P_j} \right)^{\frac{\gamma-1}{\gamma}} \right)} \quad (3)$$

$$\sum dm_i = dm_{leak} \quad (4)$$

Where S_i is the section, and γ the specific heat ratio. In a first approach no leaks due to imperfect sealing are considered so $\sum dm_{leak} = 0$.

The differential system is solved in time, volume variation being an entry of the model that is time (or crank angle) dependant. Due to the generic model, different kinematics for volume dependency on time can be tested in the perspective of optimization and evaluation of different engine or cycle configurations.

Model closure is carried by the calculation of different terms, see Fig. 2.

- $V = f(t, N)$ and $dV = f(t, N)$ depend on the geometry of the engine
- $(cv, h, \gamma) = f(T)$ are calculated with the Janaf table [19]

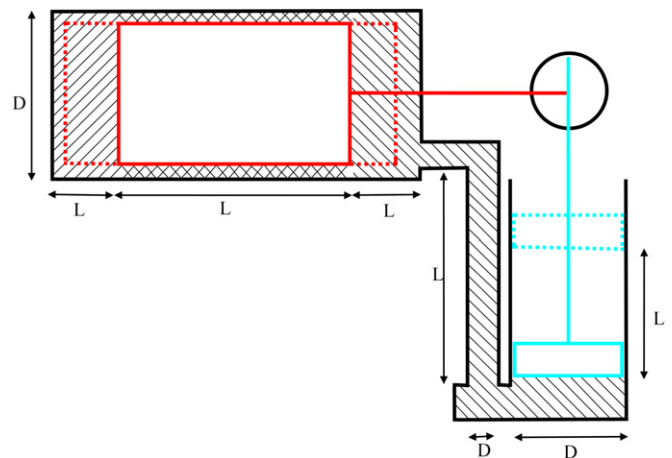


Fig. 1. Various zones of the model.

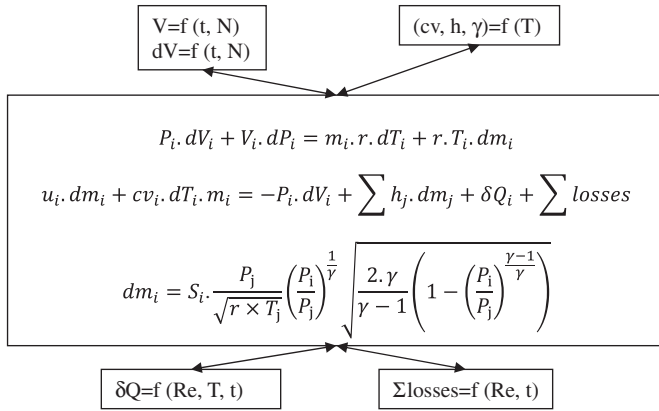


Fig. 2. Interaction between different parameters.

- $\delta Q = f(Re, T, t)$ is calculated from the Newton's law [20,21] and depends on exchange areas which depend on pistons positions, it is calculated for each internal wall
- $\Sigma losses = f(Re, t)$ is the whole of losses we could want to take into account, like possible gas leak

Where N is the engine speed, t the time and Re the Reynolds number.

For the calculation of the heat exchange coefficient we use the equation of Colburn [21], equation (5). To calculate an approximate corrected Nusselt number a function of the ratio L/D of each space is used as a corrective term, equation (6), it allows calculation of the heat exchange coefficient, equation (7) [22].

$$Nu_D = 0.023 \times Re_D^{0.8} \times Pr^{1/3} \quad (5)$$

$$Nu_{cor} = Nu_D \cdot \left(1 + f\left(\frac{D}{L}\right) \right) \quad (6)$$

$$Nu_{cor} = \frac{hconv \cdot D}{\lambda} \quad (7)$$

Nu , Re and Pr are respectively the Nusselt, Reynolds and Prandtl numbers, D and L are respectively the diameter and the length of the volume, λ is the conductive coefficient of the gas and $hconv$ is here the convective coefficient. For the calculation of Re_D , D is used as characteristic length equation (8).

$$Re_D = \frac{v \cdot D}{\nu} \quad (8)$$

Where ν is the cinematic viscosity and v is the average gas speed calculated from equation (9).

$$v = \frac{N}{60} \cdot 2 \cdot stroke \cdot \left(\frac{D_{ref}}{D} \right)^2 \quad (9)$$

Where D_{ref} is the cylinder bore or equivalent characteristic length of the volume containing the gas and stroke is the one of the both piston and displacer. In the equation (9) we used the mean velocity of the piston because the calculation of the instantaneous speed enlarges a lot the calculation time to a light improvement of the result.

Like we are in alternative flow, this method gives only an approximate convective coefficient, but with an empirical coefficient it allows good concordance.

From the calculation of pressures, the model calculates numerical work equation (10), power equation (11) and efficiency equation (13) of the engine. It is therefore possible to evaluate the

performance of a given set of parameters (exchange coefficients, kinematics, and dimensions) on the trends of performance of an engine. Note that no specific geometry for the engine is adopted a priori, which allows testing different configurations with specific kinematic or different kind of gases. The working conditions, heat sources, speed, are also adjustable in function of need.

$$W = \sum (P_c \cdot dV_c + P_h \cdot dV_h) \quad (10)$$

$$Pw = W \times \frac{N}{60} \quad (11)$$

$$Q_{in} = A_h \cdot hconv_h \cdot (T_{wh} - T_h) \quad (12)$$

$$\mu = \frac{W}{Q_{in}} \quad (13)$$

3. Experiments

3.1. Experimental rig

Tests have been carried out with a GT03 Stirling engine; see Fig. 3, which has been modified in order to allow various configurations and measurements.

3.1.1. Engine modifications

The first modification on the engine was to optimize the heat transfers, Fig. 4. For the hot and cold side, the original parts were replaced with metal parts with good thermal conductance: copper for the hot one and aluminium for the cold one. Between these, an insulating material, Duratec®, with a thermal conductivity of 0.4 W/(m K) and which withstands to 1000 °C, has been placed.

In order to test the engine in various configurations, other modifications were realized making it more flexible. For that purpose a wedge was installed between the copper and the insulating material. The length of the wedge, see Fig. 5 and Fig. 6, allows to modify the dead volume of the hot cylinder and so the volumetric ratio of the engine.

Finally some machining was realized for instrumentation. In each side, see Fig. 7 and Fig. 8, holes were threaded for the gas pressure and temperature sensors and one non emerging hole was made for wall temperature sensors. On the flywheel, see Fig. 9, three holes were made to fix the cog wheel which allows the angular position measuring.

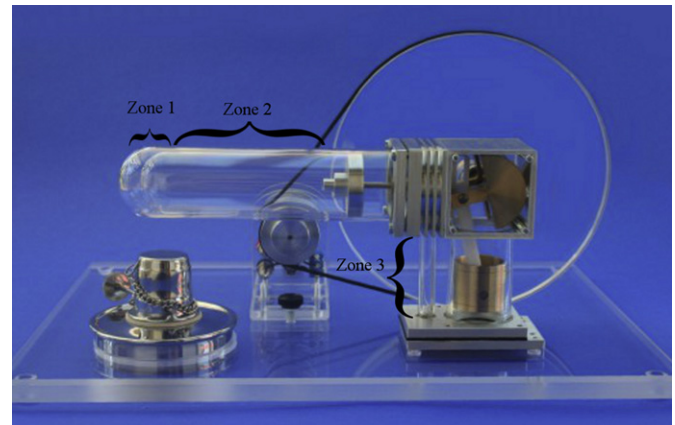


Fig. 3. Initial engine.

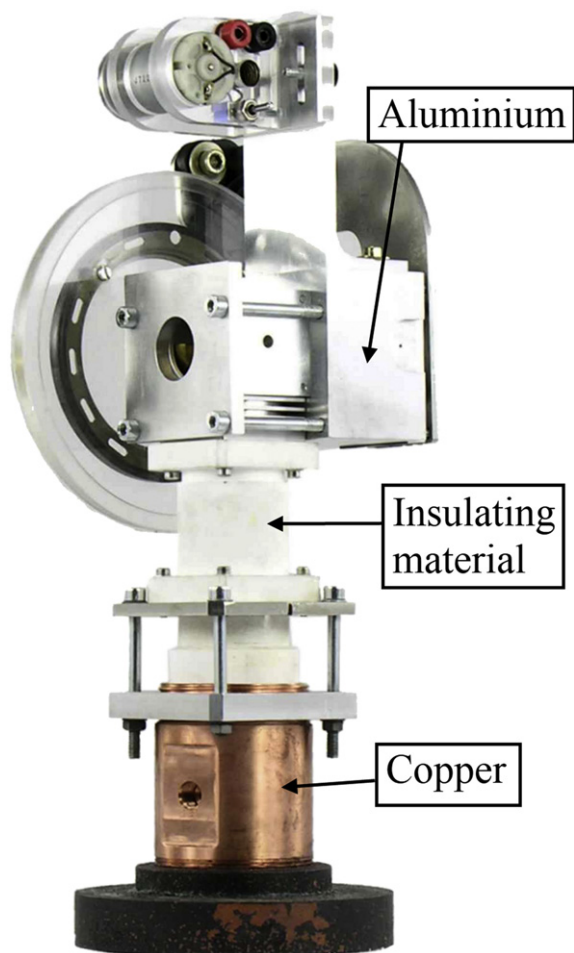


Fig. 4. Modified engine.

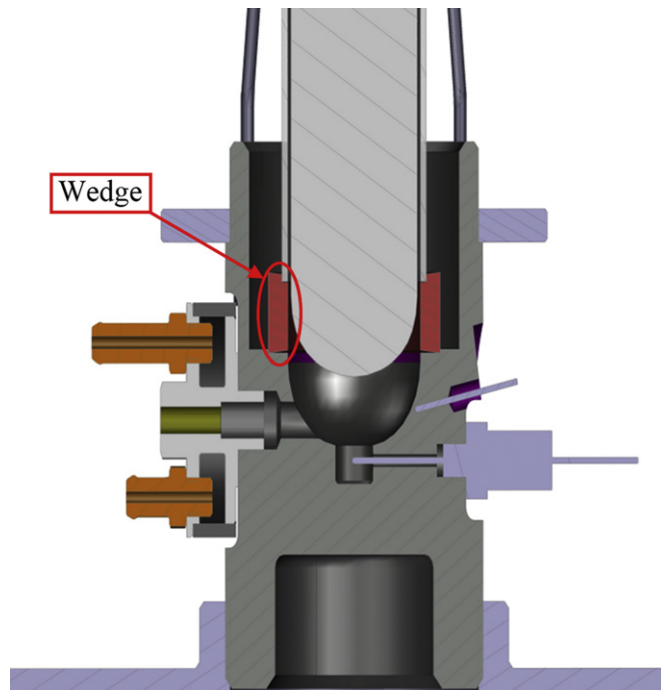


Fig. 6. Wedge length = 12 mm.

3.1.2. Sensors

Piezoelectric pressure sensors with resonance frequency higher than 200 kHz and measuring slot of 0–3.5 bar are used. Thermocouples of type K with a diameter of 1 mm are used for measuring the temperature. A Hall effect sensor coupled to a cog wheel of sixty (less two for TDC positioning) teeth are used to detect the crank angle position.

3.2. Test conditions

Tests have been realized with ambient temperature and pressure. They represent the initial conditions of the gas. During the tests the heat sink is realized by water circulation around the cylinder and the heat source is realized by a butane/propane gas-burner. Speed and load are regulated by an electrical engine. The volumetric ratio is adjustable by changing the wedge used.

Three engine parameters can therefore be adjusted. The volumetric ratio, on the cold side is constant at 2.64 and on the hot side it can be changed from 2.67 to 13.38, the

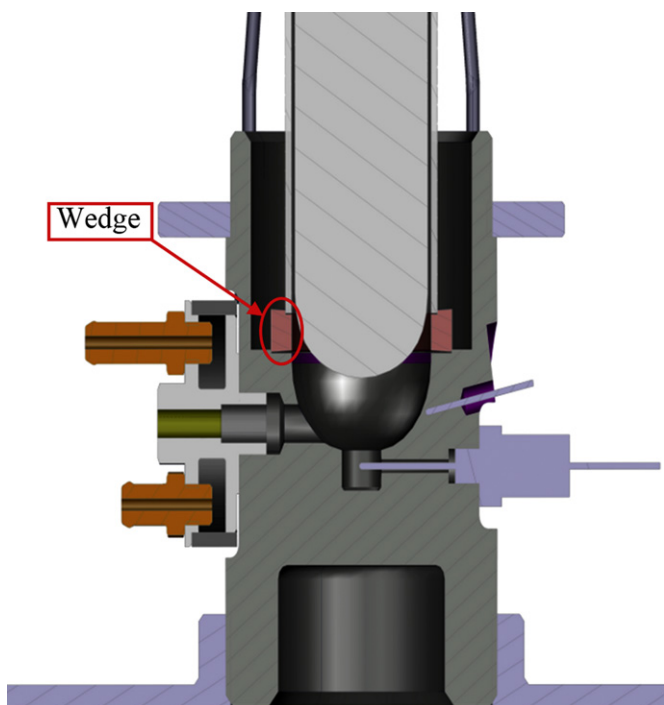


Fig. 5. Wedge length = 4 mm.

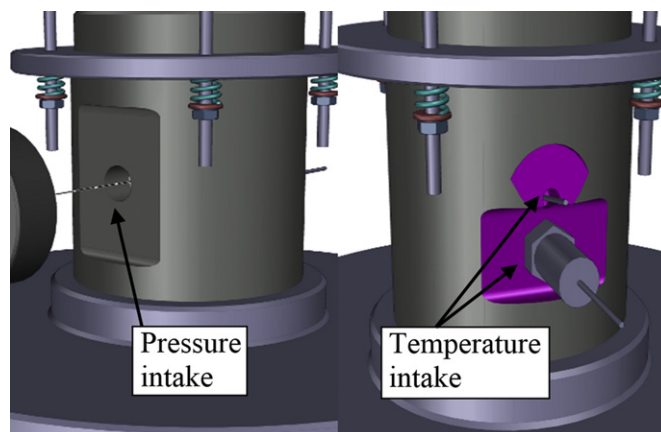


Fig. 7. Hot side.

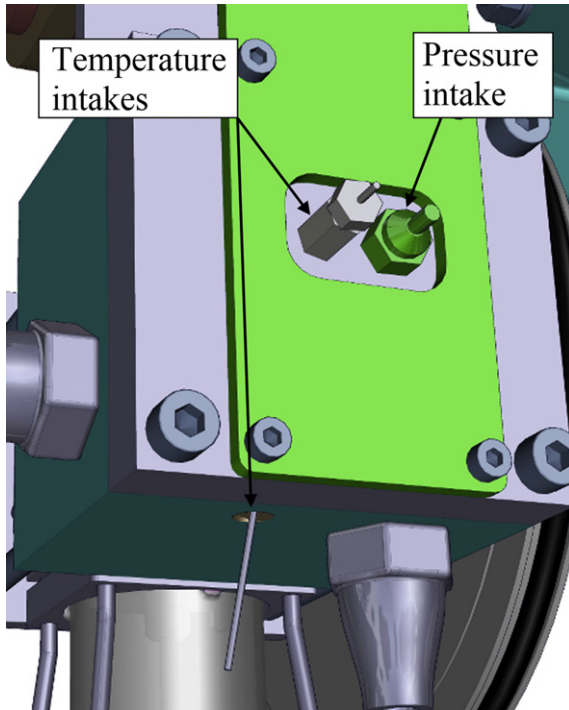


Fig. 8. Cold side.

temperature on the hot side (varies from 50 °C to 550 °C) and the speed of the engine (varies from 300 rpm to 2100 rpm). Tests are carried for various combinations of these parameters, see Fig. 10. Engine performance is calculated from pressure measurements equation (10). Volume is deduced from CAD files, see Fig. 11.

4. Results and discussion

Pressure measurements and corresponding calculations are presented for different hot source temperatures and engine speeds, with

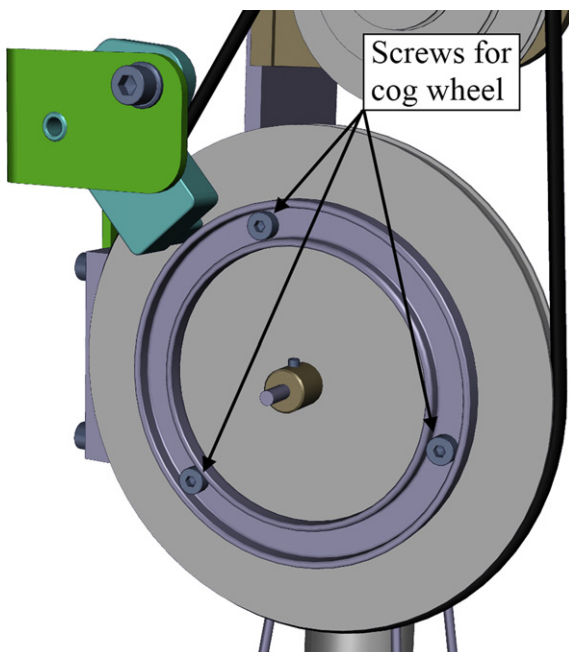
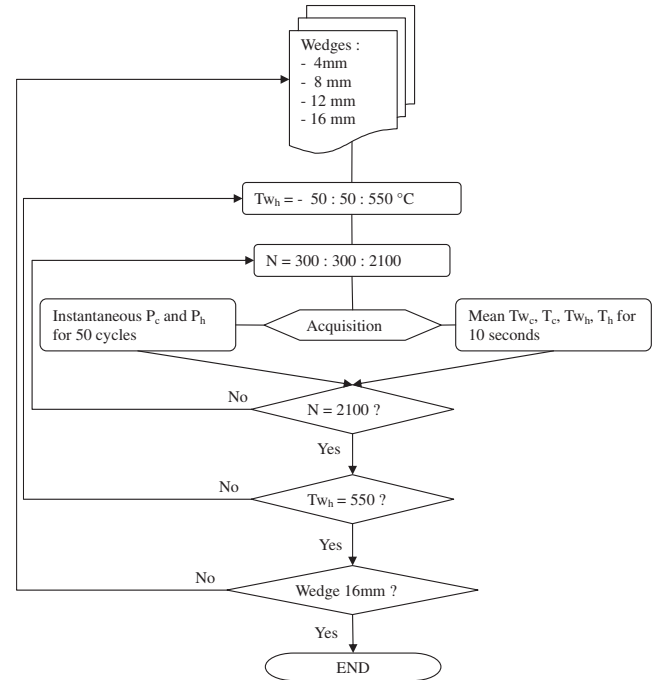


Fig. 9. Cog wheel fixation.



Where T_{wc} , T_c , T_{wh} , T_h are the wall and gas, cold and hot temperature in °C, P_c and P_h the cold and hot absolute pressure in Pa and N the engine speed in rpm.

Fig. 10. Step of experimentation.

the wedge of 8 mm. Following, power values for different parameter changes are shown.

Fig. 12(a) shows pressure evolution at 100 °C and for 300 rpm. Pressure levels oscillate around ambient pressure showing relatively good agreement between experiments and modelling. By increasing speed Fig. 12(b) and temperature Fig. 12(c) or both Fig. 12(d) slight differences can be observed although the trends of variation are correctly predicted by modelling. When a factor of six is applied to engine speed (namely from 300 rpm to 1800 rpm) the predicted

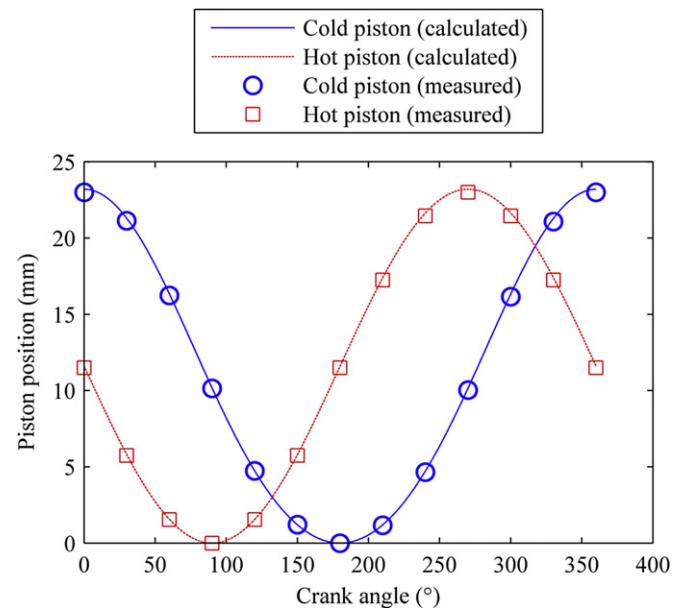


Fig. 11. Pistons positions in function of the crank angle.

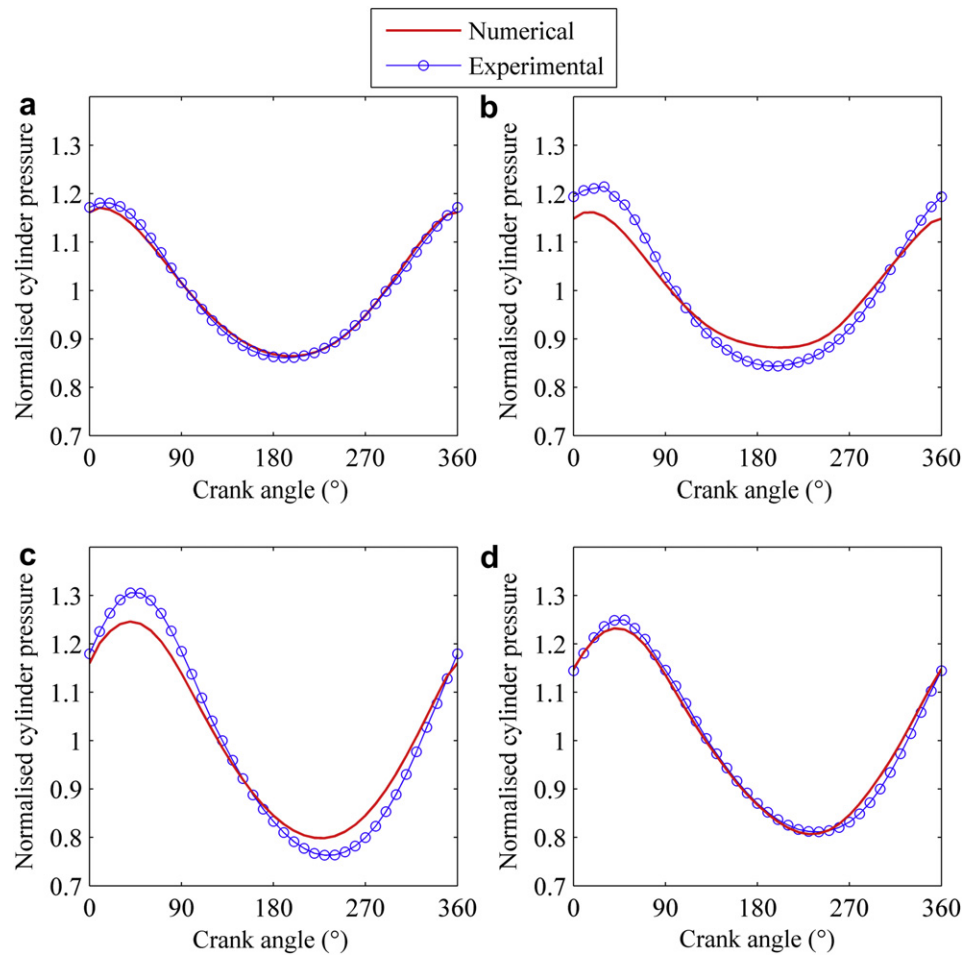


Fig. 12. Experimental and numerical pressure for (a) 100 °C, 300 rpm, (b) 100 °C, 1800 rpm, (c) 550 °C, 300 rpm, (d) 550 °C, 1800 rpm.

values given by the model follow correctly the evolutions observed in experiments. An increase of 450 °C in temperature is also efficiently modelled and little differences with experiments can be observed. Hence, the ability of the model to predict pressure evolutions for large ranges of functioning parameters in the scaled engine is demonstrated.

Fig. 13 shows the evolution of the experimental and numerical power as a function of the engine speed with the wedge of 4 mm and a hot temperature of 300 °C. The ability of the model to predict engine performance trends with speed is good in a range covering 300–2100 rpm.

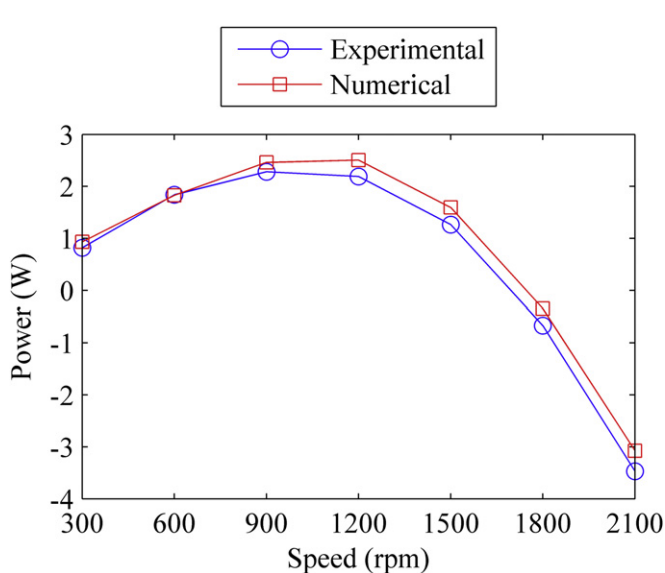


Fig. 13. Power evolution in function of the speed.

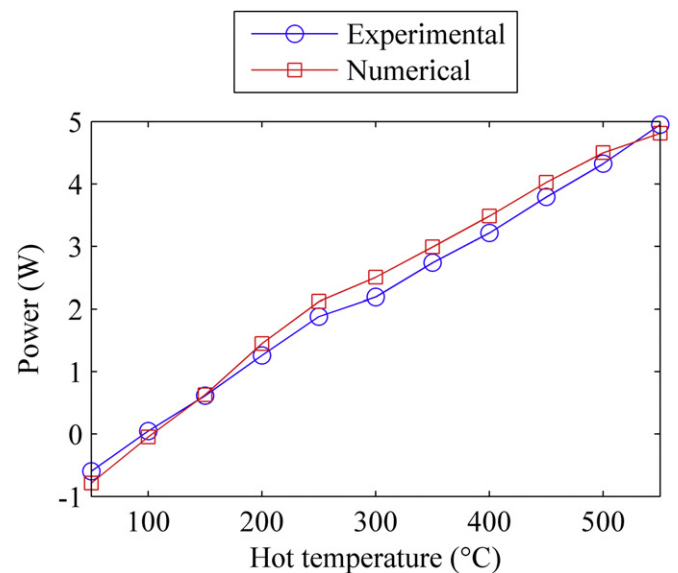


Fig. 14. Power evolution in function of the hot temperature.

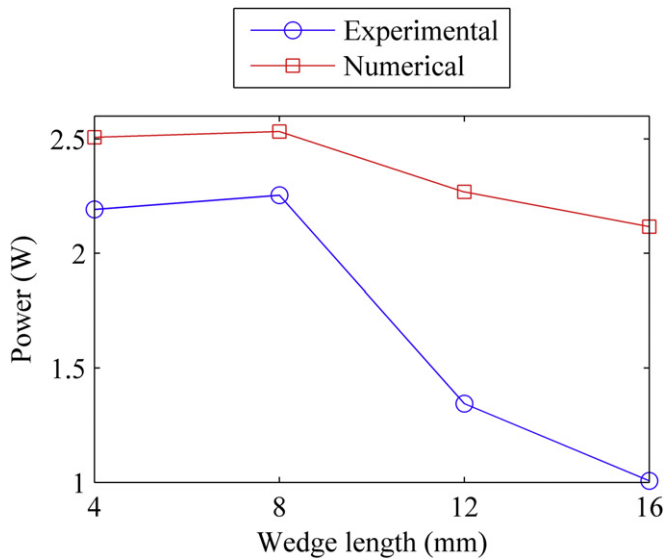


Fig. 15. Power evolution in function of the wedge length.

Fig. 14 shows the evolution of the experimental and numerical power as a function of the hot temperature with the wedge of 4 mm and an engine speed of 1200 rpm. The ability of the model to predict engine performance trends with temperature is good in a range of 50–550 °C.

Fig. 15 shows the evolution of the experimental and numerical power as a function of the wedge length with a speed of 1200 rpm and a hot temperature of 300 °C. Again, the ability of the model to predict engine performance trends with different geometrical configurations is also acceptable.

These figures underline that the model can reproduce rather precisely the behaviour of the experimental engine with regard to the variation of the parameters, engine speed, hot temperature and volumetric ratio. Fig. 16 shows the efficiency/power plot of the scaled engine for an engine speed range of 10–10,000 rpm, with a 350 °C and 600 °C hot source and 8 mm wedge. The characteristic plot of a Novikov–Chambadal [16,23,24] engine with finite rate heat transfer and heat leak losses can be observed.

One can expect that the validity of calculations remain when extrapolating to other operating conditions but also to other

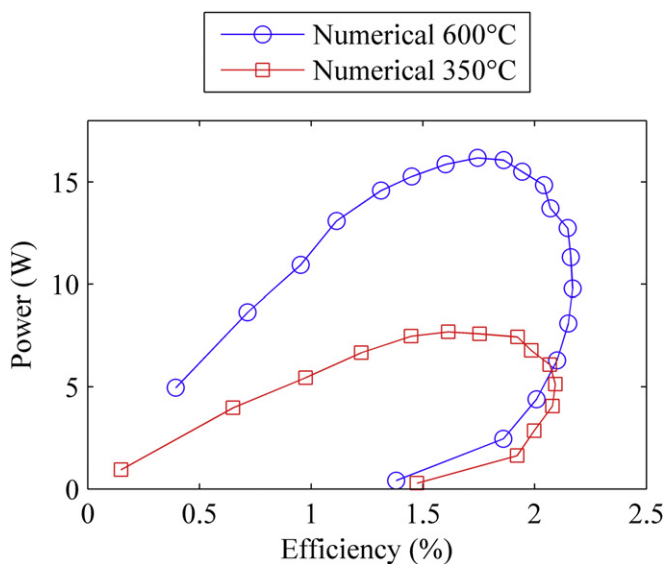


Fig. 16. Efficiency/power plot.

dimensions of a similar engine. As long as the expressions of heat and mass transfer remain valid, one can expect that at least the trends for engine performance are well predicted by the model. For very high temperatures and higher levels of pressure for instance, radiation and convection terms might change. Also, for different geometrical configurations of engine, conductive losses and leaks might be different. Nevertheless, by knowing the nature and value of coefficients in such terms it is possible to predict engine performance. Also, it is possible to estimate the gain by changing the value of any of such parameters or engine dimensions, allowing cost/performance studies prior to machining prototypes.

5. Conclusion

In this article we present a numerical zero dimensional, three zones model of a Stirling engine with time-dependant heat and mass exchanges and its confrontation to experimental results. The model shows a good concordance with the experimental trends over the entire range of parameters tested. More than 300 different configurations of running conditions were tested covering a range of hot temperatures from 50 to 550 °C, rotational speeds of 300–2100 rpm and changes in volume ratio from 2.67 to 13.38. It brings out that the model gives correct trends and values about the influence of various working parameters on the engine performances.

Moreover, the model gives detailed information about the thermodynamic state of the engine. With the present instrumentation we have been allowed to compare pressures and work, and so power, of the real engine and the model. The use of thin (dozens of μm) thermocouples will also be carried out for the comparison of temperatures and efficiency and for further refinement of modelling.

A validation of the model has been carried out on a small scale engine but for a wide operating range; it can be used then to study greater size Stirling engines and furthermore to choose engine characteristics and to optimize its performances. In the future, optimization algorithms will be implemented for the study of best set of parameters for heat exchangers, piston kinematics and engine configuration.

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